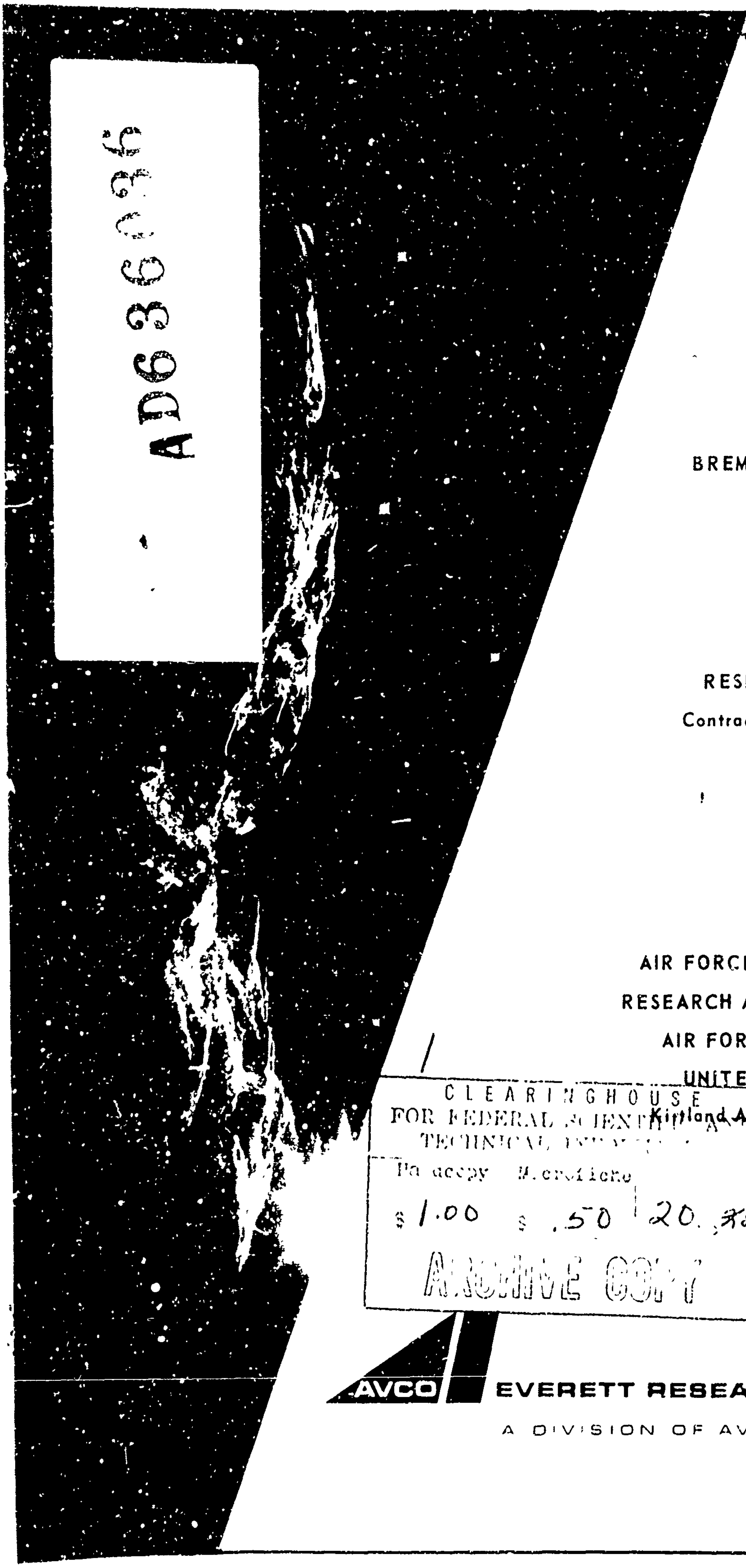

BREMSSTRAHLUNG IN AIR

B. Kivel

RESEARCH REPORT 249

Contract No. AF 29(601)-7055

Project No. 5710

June 1966

prepared for

AIR FORCE WEAPONS LABORATORY

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AIR FORCE SYSTEMS COMMAND

UNITED STATES AIR FORCE

Kirtland Air Force Base, New Mexico

EVERETT RESEARCH LABORATORY

A DIVISION OF AVCO CORPORATION

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*Submitted to Journal of Quantitative and Radiation Spectroscopy Transfer.

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ABSTRACT

The partial wave analysis for Bremsstrahlung and elastic scattering by molecular nitrogen is reviewed. These studies suggest that the Bremsstrahlung in the field of molecular nitrogen can be estimated from the measured momentum transfer cross section. Using this approximation for the molecular components and recent calculations of Bremsstrahlung in the field of atomic oxygen and nitrogen, we estimate the emissivity and intensity from neutral Bremsstrahlung in high temperature air.

I. Introduction

In air 8000°K and 1 atmosphere of density the Bremsstrahlung from electrons accelerated in the fields of neutral atoms and molecules contributes about 10% of the total (spectrally integrated) emissivity.[1,2,3,4] Recent measurements of infrared radiation from air and nitrogen by Taylor[5] have demonstrated that molecular nitrogen is the most important source of this radiation having about twice the cross section of that of atomic nitrogen.

Unfortunately, the molecular nitrogen which is the most important source of Bremsstrahlung high temperature air, is the most difficult to handle theoretically.

In Sect. II we outline the derivation of the quantum mechanical calculation of Bremsstrahlung from a spherically symmetric potential. In Sect. III we give a simple approximation relating the Bremsstrahlung to the momentum transfer cross section. In Sect. IV we review a calculation by Stier[6] of the elastic scattering of electrons by molecular nitrogen. Using a similar approach for Bremsstrahlung we demonstrate that the relation between Bremsstrahlung and elastic scattering is as reasonable in the molecular as in the atomic case. In Sect. V we give a summary of our estimates of neutral Bremsstrahlung in high temperature air. In addition to the molecular nitrogen we give results for molecular oxygen using the same approximation given in Sect. III and justified in Sect. IV for molecular nitrogen. The atomic oxygen and nitrogen results are based on recent calculations designed to be consistent with the photo-absorption cross section of negative atomic oxygen.[7]

II. Bremsstrahlung from Low Energy Electrons Scattered by a Spherically Symmetric Potential

Following Nedelsky[8] we assume that the radiation is proportional to the electron acceleration matrix element squared in an analogy with the classical result

$$J = \frac{4e^2}{3c^3} \left| \underset{\sim}{a}\, E, E' \right|^2 \qquad (1)$$

Nedelsky, who uses a cut-off Coulomb potential, has the acceleration in the form

$$\underset{\sim}{a} = - \frac{dV}{dr} \underset{\sim}{j} = - \frac{Ze^2}{mr^2} \underset{\sim}{j} \qquad \text{for } r < a \qquad (2)$$

The initial electron state is determined such that when a unit incident plane wave is subtracted only the outgoing (e^{+ikr}) waves remain. The final states are composed of both in-going and out-going waves and are normalized so that each component represents one encounter per unit of time.

Although Nedelsky treats this problem in general, we will limit our review to the lowest order waves (s and p waves). We have then for the initial state

$$\psi_i \doteq \frac{a_0}{\sqrt{v_i}\;\kappa_i} \left(e^{i\delta_0} G_{0i} + 3ie^{i\delta_1} \cos\theta\, G_{1i} \right) \qquad (3)$$

To obtain the total radiation, we have to sum over final states and to this low order approximation we have for final states

$$(\psi_0)_f = \frac{2}{\sqrt{v_f}} \frac{1}{\sqrt{4\pi}} G_{0f}$$

$$(\psi_{1,0})_f = \frac{2}{\sqrt{v_f}} \sqrt{\frac{3}{4\pi}} \cos\theta\, G_{1f} \qquad (4)$$

$$(\psi_{1,\pm 1})_f = \frac{2}{\sqrt{v_f}} \sqrt{\frac{3}{8\pi}} \sin\theta\; e^{\pm i\phi}\, G_{1f}$$

Since the radial vector, $\underset{\sim}{j}$, is

$$\underset{\sim}{j} = \underset{\sim}{i}_x \sin\theta \cos\phi + \underset{\sim}{i}_y \sin\theta \sin\phi + \underset{\sim}{i}_z \cos\theta \tag{5}$$

the radiation is given by

$$J = \frac{64\pi e^6 Z^2}{3c^3 m^2 v_i v_f \kappa_i^2} \left\{ \left| \int_0^a G_{0i} G_{1f} \frac{1}{r^2} dr \right|^2 + \left| \int_0^a G_{1i} G_{0f} \frac{1}{r^2} dr \right|^2 \right\} \tag{6}$$

The emission cross section is given by

$$Q_e = J/h\nu \tag{7}$$

and using the detailed balance relation between the absorption and emission cross section

$$Q_a = \left(\frac{c^2 v_i^2}{8\pi\nu^2 v_f} \right) Q_e \tag{8}$$

We find the absorption cross section to be

$$Q_a = \frac{256\pi^2}{3} \frac{\alpha a_o^5}{\kappa_i \kappa_f^2 (\Delta\kappa^2)^3} \left\{ \left| \int_0^a G_{0i} G_{1f} \frac{Z}{r^2} dr \right|^2 + \left| \int_0^a G_{1i} G_{0f} \frac{Z}{r^2} dr \right|^2 \right\} \tag{9}$$

This is just the expression used by Chandresakar and Breen[9] and others where Z/r^2 is the radial derivative of the potential energy in atomic units.

III. Hundley's Result (Born Approximation)

The close relation between the Bremsstrahlung and the elastic scattering cross sections is seen in the earliest quantum mechanical studies of this problem. Nedelsky,[8] using a cut-off Coulomb potential, found the same maxima and minima as Allis and Morse[10] in the elastic scattering. Thus, the same minima which explained the Ramsauer minimum cross section in argon and krypton indicate in Nedelsky's quantum mechanical calculation that the Bremsstrahlung from low energy electrons will be small for these elements. More recently, Ohmura and Ohmura[11] have related the hydrogen atom Bremsstrahlung to its phase shifts. The validity of this approach for hydrogen has been definitized by the recent calculations of Geltman.[12] Hundley[13] has recently assumed that this may be a generally useful approach and has tried to apply the Born approximation to this problem.

Hundley's relation for the Bremsstrahlung emission is (in the limit $\kappa_i = \kappa_f \equiv \kappa$)

$$\frac{d\sigma}{d\omega} = \frac{16\, e^2 \hbar \kappa^2}{3m^2 \omega c^3} \quad \frac{Q_m}{4\pi a_o^2} \qquad (10)$$

and Q_m is the momentum transfer cross section. The intensity (J) of our previous section is defined by

$$J = 2\pi h\nu \frac{d\sigma}{d\omega} \qquad (11)$$

In applying this result, Hundley suggested using the zero energy scattering length to determine the momentum transfer cross section. Since the low energy nitrogen cross section is apparently fairly small[4] this extension would imply a small molecular nitrogen Bremsstrahlung. It seems to us more reasonable to retain the original momentum transfer cross section in Eq. (10) and thereby avoid the question of the low energy limit and extrapolating to the energies of interest.

IV. Molecular Nitrogen

A reasonably successful attempt at predicting the elastic scattering of low energy electrons by molecular nitrogen has been given by Stier.[6] Although the model for the electron molecule interaction is crude, it is adjusted on a semi-empirical basis to give the scattering resonance at 2.3 ev so that reasonable agreement is obtained with both the total experimental cross section and its angular distribution. A significant difference between this case and the spherically symmetric one is that the p-wave has two parts depending on the projection of its angular momentum along the internuclear axis of the molecule. This projection is designated by m. There are two important states with ℓ, m values p, σ and p, π. Stier's work indicates that it is the p, σ wave which has a resonance at 2.3 ev.

Stier gives an expression for the elastic scattering cross section in terms of the partial wave phase shifts. It is

$$Q = \frac{4\pi a_o^2}{\kappa^2} \sum_{m \leq \ell} (2-\delta_{0m}) \sin^2 \delta_{\ell, m} \tag{12}$$

where δ_{0m} is zero except when m equals 0, in which case it is 1. If the phase shift for the two p-waves is the same, then this reduces to the spherically symmetric result $[(2\ell+1) \sin^2 \delta_\ell]$.

In the case of interest for the p-waves, the m = 1 phase shift is negligible, and the remaining contribution to the cross section (m = 0) has a coefficient which is 1/3 of that for the p-wave in spherically symmetric scattering.

Following Stier we look for a corresponding effect in the Bremsstrahlung relations. Stier has solved the molecular scattering problem as an expansion in the parameter $\epsilon = d/\lambda$ where d is the nuclear separation in the molecule and λ is $\hbar/mv$, the deBroglie wavelength for the electron.

In the limit of ϵ going to zero, the wave functions are very similar to those we encountered in the symmetrical scattering estimate of Bremsstrahlung. The one exception is that the radial part of the p-wave function

now has two parts corresponding to the value of m. We have, for the initial state,

$$\psi_i \doteq \frac{a_o}{\kappa_i \sqrt{v_i}} \left[e^{i\delta_0} (G_0)_i + 3ie^{i\delta_1} (G_{10})_i \right] \tag{13}$$

where we have chosen the z-axis along the direction of the incident plane wave so that only $m = 0$ occurs. For the final states (as in Sect. II) we again consider only the four lowest terms which are

$$(\psi_0)_f = \frac{2}{\sqrt{v_f}} \frac{1}{\sqrt{4\pi}} (G_0)_f$$

$$(\psi_{1,0})_f = \frac{2}{\sqrt{v_f}} \sqrt{\frac{3}{4\pi}} \cos\theta \, (G_{10})_f \tag{14}$$

$$(\psi_{1,\pm 1})_f = \frac{2}{\sqrt{v_f}} \sqrt{\frac{3}{8\pi}} \sin\theta \, e^{\pm i\phi} (G_{11})_f$$

where $(G_{\ell m})_f$ is the radial function for the ℓ, m wave. Substituting these wave functions into Nedelsky's expression for the intensity, we obtain

$$J = \frac{16\pi}{v_f v_i \kappa_i^2} \left\{ \left| \int_0^\infty (G_{10})_i (G_0)_f \frac{dV}{dr} dr \right|^2 + \frac{1}{3} \left| \int_0^\infty (G_0)_i (G_{10})_f \frac{dV}{dr} \right|^2 \right.$$

$$\left. + \frac{2}{3} \left| \int_0^\infty (G_0)_i (G_{11})_f \frac{dV}{dr} \right|^2 \right\} \tag{15}$$

As in the elastic scattering, if only the $m = 0$ phase of the $\ell = 1$ states is important, the Bremsstrahlung is reduced by a factor 1/3.

When the incident plane wave is at an angle α with the z-axis, we obtain the same result so that in this approximation the average over angles of incidence gives the same result as taking the incident wave along the z-axis.

We conclude to this order of approximation that the Bremsstrahlung is proportional to the momentum transfer cross section, even in the non-spherically symmetric case.

V. Results and Summary

We conclude, as a result of 1) the above study of Bremsstrahlung for molecular nitrogen, 2) the Born approximation as given by Hundley, and 3) our numerical calculations of the atomic oxygen and atomic nitrogen neutral Bremsstrahlung and elastic scattering cross sections,[7] that Hundley's approximation should give a reasonable estimate of the Bremsstrahlung absorption cross section.

Using this approximation we can now give estimates of the neutral Bremsstrahlung from high temperature air. The emissivity per unit length is determined according to

$$\frac{\epsilon}{L} = 2N_i N_e Q_a \tag{16}$$

where the density of particles N_i and N_e are taken from the Bureau of Standards results by Hilsenrath et al.[15] The radiation absorption cross section Q_a is determined from the Maxwell averaged momentum transfer cross section Q_m by

$$Q_a = 2.9 \times 10^{-19} Q_m \left(\frac{1.36^{-2}\lambda}{1.24\mu}\right)^3 \frac{4}{\sqrt{\pi}} \left(\frac{T}{11{,}600 \times 0.544}\right)^{3/2} \tag{17}$$

For the Maxwell average we have used

$$Q_m = \frac{2}{\sqrt{\pi}} \int_0^\infty Q\left(E + \frac{1.24\mu}{2\lambda}\right)\sqrt{u}\; e^{-u}\, du \tag{18}$$

where $u = E/kT$ and E is the initial electron energy. We have used measured values of Q and by taking values at $E + (1.24/2\lambda)$ we have attempted to take account of the fact that the final state of the electron is at higher energy than the initial state.

With the emissivity per unit length we can calculate the spectral intensity according to

$$I_\lambda = \frac{\epsilon}{L} \frac{\sigma T^4}{2\pi 6} \frac{u^4 e^{-u}}{\lambda} \tag{19}$$

where $u = h\nu/kT$.

We use the results given by Phelps et al.[14] for the momentum transfer cross section of molecular nitrogen and of Ramsauer et al.[16] for molecular oxygen. For atomic oxygen and nitrogen we go directly from our calculation of the Bremsstrahlung. The results are given in four tables. These contain the intensity (I) in watts/cm^3/ster micron and the emissivity/cm (ϵ/L), i.e., the fraction of the blackbody intensity from 1 centimeter of optical depth. These results are given for temperatures between 3000 and 15,000°K, densities between 10^{-3} and 10 atm and wavelengths between 0.3 and 4.8 μ. A sample of these data are plotted in Fig. 1 which gives the intensity for the four neutral species considered as a function of wavelength at $T = 9000$°K and $\rho/\rho_o = 10$. Also shown is the blackbody limit.

These results are consistent with the measurements of Taylor[5] and the tables of emissivity prepared by Allen[17] based on Taylor's measurements.

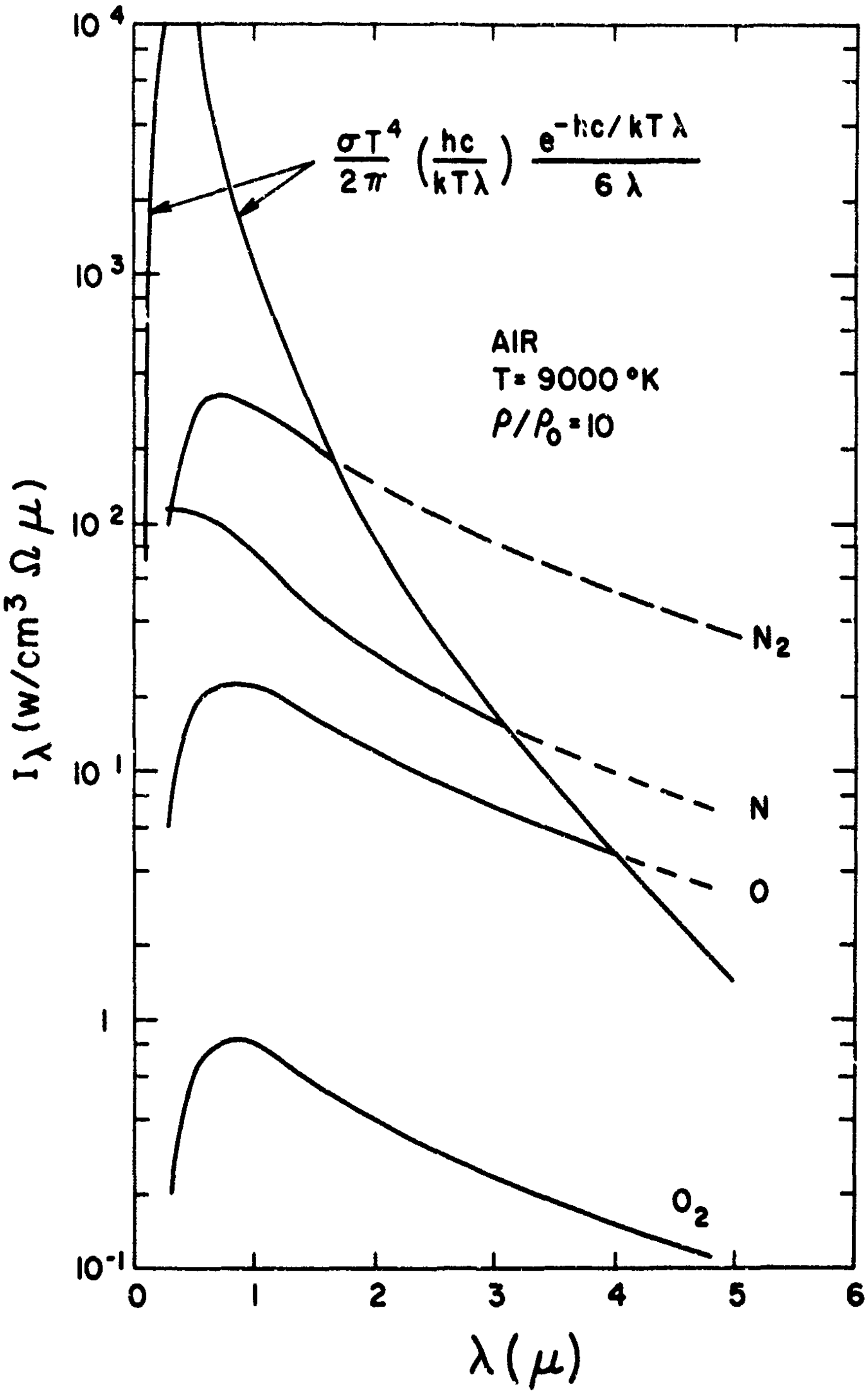

Fig. 1 The spectral intensity in watts per cubic centimeter-steradian-micron as a function of wavelength in microns. The Bremsstrahlung continuum components for molecular N_2 and O_2 and atomic N and O are given separately. Also shown is the corresponding Planck limit. These curves are for a temperature of 9000°K and a density ρ/ρ_0 = 10. The component curves are shown as dashed lines when the intensity from 1 cm of optical depth exceeds the Planck limit.

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TABLE I

Bremsstrahlung absorption cross section (Q_a), emissivity per centimeter (ϵ/L) and spectral intensity (I_λ) in watts/cm^3 steradian micron for molecular nitrogen.

T(°K)	λ(μ)	Q_a(cm^5) x 10^{40}	ρ/ρ$_o$ = 0.001		= 0.01		= 0.1		= 1.0		= 10.0	
			ε/L	I_λ	ε/L	I_λ	ε/L	I_λ	ε/L	I_λ	ε/L	I_λ
3000	0.3	1.91E-01	4.44E-15	1.32E-15	1.66E-13	4.93E-14	5.52E-12	1.64E-12	1.73E-10	5.13E-11	5.16E-09	1.53E-09
	0.6	7.79E-01	1.80E-14	4.99E-13	6.76E-13	1.87E-11	2.24E-11	6.20E-10	7.03E-10	1.94E-08	2.10E-08	5.80E-07
	1.2	5.00E 00	1.16E-13	5.46E-12	4.34E-12	2.04E-10	1.44E-10	6.79E-09	4.51E-09	2.13E-07	1.35E-07	6.35E-06
	2.4	3.88E 01	8.99E-13	9.78E-12	3.37E-11	3.66E-10	1.12E-09	1.22E-08	3.50E-08	3.81E-07	1.04E-06	1.14E-05
	4.8	2.59E 02	6.93E-12	6.41E-12	2.59E-10	2.40E-10	8.61E-09	7.96E-09	[illegible]	2.49E-07	8.05E-06	7.45E-06
6000	0.3	4.80E-01	1.61E-11	1.42E-08	1.59E-09	1.41E-06	1.08E-07	9.56E-05	5.91E-06	5.23E-03	2.46E-04	2.18E-01
	0.6	2.74E 00	9.19E-11	1.39E-07	9.08E-09	1.37E-05	6.16E-07	9.30E-04	3.37E-05	5.09E-02	1.40E-03	2.12E 00
	1.2	1.73E 01	5.82E-10	2.03E-07	5.75E-08	2.00E-05	3.90E-06	1.36E-03	2.14E-04	7.44E-02	8.89E-03	3.10E 00
	2.4	1.28E 02	4.30E-09	1.27E-07	4.25E-07	1.26E-05	2.88E-05	8.53E-04	1.58E-03	4.67E-02	6.57E-02	1.94E 00
	4.8	9.83E 02	3.30E-08	5.02E-08	3.26E-06	4.97E-06	2.21E-04	3.37E-04	1.21E-02	1.84E-02	5.04E-01	7.68E-01
9000	0.3	7.74E-01	1.26E-11	1.47E-07	3.76E-09	4.79E-05	9.18E-07	1.17E-02	1.11E-04	1.42E 00	8.01E-03	1.02E 02
	0.6	5.39E 00	8.04E-11	4.60E-07	2.62E-08	1.50E-04	6.39E-06	3.65E-02	7.75E-04	4.43E 00	5.57E-02	3.19E 02
	1.2	3.62E 01	5.41E-10	3.67E-07	1.76E-07	1.19E-04	4.30E-05	2.91E-02	5.21E-03	3.54E 00	3.75E-01	2.54E 02
	2.4	2.69E 02	4.01E-09	1.66E-07	1.31E-06	5.39E-05	3.19E-04	1.32E-02	3.87E-02	1.60E 00	2.78E 00	1.15E 02
	4.8	2.07E 03	3.09E-08	5.56E-08	1.01E-05	1.81E-05	2.45E-03	4.42E-03	2.98E-01	5.36E-01	2.14E 01	3.85E 01
12000	0.3	1.07E 00	2.81E-12	1.36E-07	1.87E-09	9.03E-05	7.59E-07	3.66E-02	2.37E-04	1.14E 01	4.05E-02	1.95E 03
	0.6	[illegible]	2.17E-11	2.42E-07	1.45E-08	1.61E-04	5.87E-06	6.54E-02	1.83E-03	2.04E 01	3.13E-01	3.49E 03
	1.2	5.07E 01	1.54E-10	1.46E-07	1.02E-07	9.70E-05	4.16E-05	3.94E-02	1.30E-02	1.23E 01	2.22E 00	2.10E 03
	2.4	4.42E 02	1.16E-09	5.66E-08	7.73E-07	3.77E-05	3.14E-04	1.53E-02	9.79E-02	4.77E 00	1.67E 01	8.16E 02
	4.8	3.43E 03	9.00E-09	1.76E-08	5.99E-06	1.17E-05	2.43E-03	4.76E-03	7.59E-01	1.49E 00	1.30E 02	2.54E 02
15000	0.3	1.37E 00	8.83E-14	9.49E-09	4.48E-10	4.82E-05	4.25E-07	4.57E-02	2.04E-04	2.19E 01	[illegible]	7.60E 03
	0.6	1.13E 01	7.24E-13	1.20E-08	3.68E-09	6.11E-05	3.48E-06	5.79E-02	1.67E-03	2.78E 01	5.80E-01	9.65E 03
	1.2	8.28E 01	5.32E-12	6.16E-09	2.70E-08	3.13E-05	2.56E-05	2.96E-02	1.23E-02	1.42E 0[illegible]	4.26E 00	4.93E 03
	2.4	6.33E 02	4.07E-11	2.20E-09	2.07E-07	1.11E-05	1.96E-04	1.06E-02	9.39E-02	5.07E 00	3.26E 01	1.76E 03
	4.8	4.94E 03	3.18E-10	6.54E-10	1.61E-06	3.32E-06	1.53E-03	3.15E-03	7.33E-01	1.51E 00	2.55E 02	5.24E 02

TABLE II

Bremsstrahlung absorption cross section (Q_a), emissivity per centimeter (ϵ/L) and spectral intensity (I_λ) in watts/cm^3 steradian micron for molecular oxygen.

$T(^\circ K)$	$\lambda(\mu)$	$Q_a(cm^5) \times 10^{40}$	$\rho/\rho_o = 0.001$		$= 0.01$		$= 0.1$		$= 1.0$		$= 10.0$	
			ϵ/L	I_λ	ϵ/L	I_λ	ϵ/L	I_λ	ϵ/L	I_λ	ϵ/L	I_λ
3000	0.3	5.29E-02	9.95E-17	2.95E-17	7.78E-15	2.31E-15	3.32E-13	9.86E-14	1.13E-11	3.35E-12	3.45E-10	1.02E-10
	0.6	4.15E-01	7.81E-16	2.16E-14	6.11E-14	1.69E-12	2.61E-12	7.20E-11	8.85E-11	2.45E-09	2.71E-09	7.49E-08
	1.2	3.17E 00	5.97E-15	2.82E-13	4.67E-13	2.20E-11	1.99E-11	9.40E-10	6.77E-10	3.19E-08	2.07E-08	9.77E-07
	2.4	2.37E 01	4.46E-14	4.85E-13	3.49E-12	3.80E-11	1.49E-10	1.62E-09	5.05E-09	5.50E-08	1.55E-07	1.68E-06
	4.8	1.77E 02	3.33E-13	3.08E-13	2.60E-11	2.41E-11	1.11E-09	1.03E-09	3.77E-08	3.49E-08	1.15E-06	1.07E-06
6000	0.3	1.47E-01	1.55E-16	1.37E-13	8.32E-14	7.36E-11	4.37E-11	3.86E-08	1.79E-08	1.58E-05	3.55E-06	3.14E-03
	0.6	1.16E 00	1.22E-15	1.84E-12	6.58E-13	9.92E-10	3.45E-10	5.21E-07	1.42E-07	2.14E-04	2.80E-05	4.23E-02
	1.2	9.03E 00	9.48E-15	3.30E-12	5.10E-12	1.78E-09	2.68E-09	9.33E-07	1.10E-06	3.83E-04	2.18E-04	7.58E-02
	2.4	6.93E 01	7.28E-14	2.15E-12	3.92E-11	1.16E-09	2.06E-08	6.08E-07	8.43E-06	2.50E-04	1.67E-03	4.94E-02
	4.8	5.33E 02	5.59E-13	8.53E-13	3.01E-10	4.59E-10	1.58E-07	2.41E-07	6.48E-05	9.88E-05	1.28E-02	1.96E-02
9000	0.3	2.70E-01	1.22E-15	1.56E-11	4.00E-13	5.09E-09	1.29E-10	1.64E-06	4.45E-08	5.66E-04	1.65E-05	2.10E-01
	0.6	2.14E 00	9.68E-15	5.54E-11	3.17E-12	1.81E-08	1.02E-09	5.85E-06	3.53E-07	2.02E-03	1.30E-04	7.47E-01
	1.2	1.67E 01	7.58E-14	5.14E-11	2.48E-11	1.68E-08	8.00E-09	5.42E-06	2.76E-06	1. 7E-03	1.02E-03	6.93E-01
	2.4	1.30E 02	5.89E-13	2.43E-11	1.93E-10	7.96E-09	6.22E-08	2.57E-06	2.15E-05	8.86E-04	7.94E-03	3.28E-01
	4.8	1.01E 03	4.59E-12	8.26E-12	1.50E-09	2.70E-09	4.84E-07	8.72E-07	1.67E-04	3.01E-04	6.18E-02	1.11E-01
12000	0.3	4.18E-01	2.74E-15	1.32E-10	1.50E-12	7.22E-08	5.71E-10	2.76E-05	1.95E-07	9.41E-03	6.12E-05	2.95E 00
	0.6	3.31E 00	2.17E-14	2.42E-10	1.18E-11	1.32E-07	4.51E-09	5.03E-05	1.54E-06	1.72E-02	4.83E-04	5.39E 00
	1.2	2.60E 01	1.70E-13	1.61E-10	9.28E-11	8.79E-08	3.54E-08	3.36E-05	1.21E-05	1.15E-02	3.80E-03	3.60E 00
	2.4	2.03E 02	1.33E-12	6.51E-11	7.27E-10	3.55E-08	2.78E-07	1.35E-05	9.48E-05	4.63E-03	2.98E-02	1.45E 00
	4.8	1.60E 03	1.05E-11	2.05E-11	5.70E-09	1.12E-08	2.18E-06	4.26E-06	7.44E-04	1.46E-03	2.33E-01	4.57E-01
15000	0.3	5.92E-01	5.33E-16	5.73E-11	1.79E-12	1.93E-07	1.29E-09	1.39E-04	5.62E-07	6.04E-02	2.12E-04	2.28E 01
	0.6	4.66E 00	4.19E-15	6.97E-11	1.41E-11	2.35E-07	1.02E-08	1.69E-04	4.42E-06	7.35E-02	1.67E-03	2.78E 01
	1.2	3.67E 01	3.30E-14	3.82E-11	1.11E-10	1.28E-07	8.01E-08	9.27E-05	3.48E-05	4.02E-02	1.31E-02	1.52E 01
	2.4	2.88E 02	2.59E-13	1.40E-11	8.73E-10	4.71E-08	6.30E-07	3.40E-05	2.73E-04	1.47E-02	1.03E-01	5.57E 00
	4.8	2.27E 03	2.04E-12	4.21E-12	6.88E-09	1.42E-08	4.96E-06	1.02E-05	2.15E-03	4.43E-03	8.14E-01	1.67E 00

TABLE III

Bremsstrahlung absorption cross section (Q_a), emissivity per centimeter (ϵ/L) and spectral intensity (I_λ) in watts/cm^3 steradian micron for atomic nitrogen.

$T(^oK)$	$\lambda(\mu)$	$Q_a(cm^5) \times 10^{40}$	ρ/ρ_o = 0.001 ϵ/L	I_λ	= 0.01 ϵ/L	I_λ	= 0.1 ϵ/L	I_λ	= 1.0 ϵ/L	I_λ	= 10.0 ϵ/L	I_λ
3000	0.3	1.75E 00	6.11E-18	1.81E-18	7.27E-17	2.16E-17	7.64E-16	2.27E-16	7.57E-15	2.25E-15	7.09E-14	2.10E-14
	0.6	3.70E 00	1.29E-17	3.57E-16	1.54E-16	4.25E-15	1.62E-15	4.46E-14	1.60E-14	4.42E-13	1.50E-13	4.14E-12
	1.2	1.80E 01	6.29E-17	2.96E-15	7.48E-16	3.52E-14	7.86E-15	3.70E-13	7.79E-14	3.67E-12	7.30E-13	3.44E-11
	2.4	1.10E 02	3.84E-16	4.18E-15	4.57E-15	4.97E-14	4.80E-14	5.23E-13	4.76E-13	5.18E-12	4.46E-12	4.85E-11
	4.8	8.00E 02	2.80E-15	2.58E-15	3.32E-14	3.07E-14	3.49E-13	3.23E-13	3.46E-12	3.20E-12	3.24E-11	3.00E-11
6000	0.3	1.75E 00	1.64E-10	1.45E-07	3.82E-09	3.38E-06	7.47E-08	6.61E-05	1.27E-06	1.12E-03	1.67E-05	1.48E-02
	0.6	3.70E 00	3.46E-10	5.23E-07	8.08E-09	1.22E-05	1.58E-07	2.38E-04	2.68E-06	4.04E-03	3.54E-05	5.34E-02
	1.2	1.80E 01	1.69E-09	5.87E-07	3.93E-08	1.37E-05	7.69E-07	2.68E-04	1.30E-05	4.54E-03	1.72E-04	6.00E-02
	2.4	1.10E 02	1.03E-08	3.05E-07	2.40E-07	7.10E-06	4.70E-06	1.39E-04	7.96E-05	2.36E-03	1.05E-03	3.11E-02
	4.8	8.00E 02	7.49E-08	1.14E-07	1.75E-06	2.66E-06	3.42E-05	5.21E-05	5.79E-04	8.83E-04	7.65E-03	1.17E-02
9000	0.3	1.75E 00	2.47E-08	3.15E-04	8.01E-07	1.02E-02	2.25E-05	2.87E-01	4.77E-04	6.07E 00	8.71E-03	1.11E 02
	0.6	3.70E 00	5.23E-08	2.99E-04	1.69E-06	9.70E-03	4.77E-05	2.73E-01	1.01E-03	5.77E 00	1.84E-02	1.05E 02
	1.2	1.80E 01	2.55E-07	1.73E-04	8.24E-06	5.59E-03	2.32E-04	1.57E-01	4.91E-03	3.33E 00	8.96E-02	6.08E 01
	2.4	1.10E 02	1.56E-06	6.42E-05	5.04E-05	2.08E-03	1.42E-03	5.85E-02	3.00E-02	1.24E 00	5.48E-01	2.26E 01
	4.8	8.00E 02	1.13E-05	2.04E-05	3.66E-04	6.60E-04	1.03E-02	1.86E-02	2.18E-01	3.93E-01	3.98E 00	7.17E 00
12000	0.3	1.75E 00	1.68E-07	8.13E-03	8.35E-06	4.03E-01	3.10E-04	1.50E 01	1.00E-02	4.85E 02	2.43E-01	1.17E 04
	0.6	3.70E 00	3.56E-07	3.97E-03	1.77E-05	1.97E-01	6.56E-04	7.31E 00	2.12E-02	2.37E 02	5.14E-01	5.73E 03
	1.2	1.80E 01	1.73E-06	1.64E-03	8.59E-05	8.13E-02	3.19E-03	3.02E 00	1.03E-01	9.78E 01	2.50E 00	2.37E 03
	2.4	1.10E 02	1.06E-05	5.16E-04	5.25E-04	2.56E-02	1.95E-02	9.52E-01	6.31E-01	3.08E 01	1.53E 01	7.46E 02
	4.8	8.00E 02	7.70E-05	1.51E-04	3.82E-03	7.47E-03	1.42E-01	2.78E-01	4.59E 00	8.99E 00	1.11E 02	2.18E 02
15000	0.3	1.75E 00	1.10E-07	1.18E-02	1.85E-05	1.99E 00	1.15E-03	1.23E 02	4.81E-02	5.17E 03	1.73E 03	1.86E 05
	0.6	3.70E 00	2.33E-07	3.88E-03	3.91E-05	6.50E-01	2.42E-03	4.03E 01	1.02E-01	1.69E 03	3.65E 00	6.07E 04
	1.2	1.80E 01	1.13E-06	1.31E-03	1.90E-04	2.20E-01	1.18E-02	1.36E 01	4.95E-01	5.72E 02	1.78E 01	2.05E 04
	2.4	1.10E 02	6.93E-06	3.74E-04	1.16E-03	6.27E-02	7.21E-02	3.89E 00	3.02E 00	1.63E 02	1.09E 02	5.85E 03
	4.8	8.00E 02	5.04E-05	1.04E-04	8.45E-03	1.74E-02	5.24E-01	1.08E 00	2.20E 01	4.52E 01	7.89E 02	1.62E 03

TABLE IV

Bremsstrahlung absorption cross section (Q_a), emissivity per centimeter (ϵ/L) and spectral intensity (I_λ) in watts/cm^3 steradian micron for atomic oxygen.

$T(^\circ K)$	$\lambda(\mu)$	$Q_a(cm^5) \times 10^{40}$	$\rho/\rho_o = 0.001$ ϵ/L	I_λ	$= 0.01$ ϵ/L	I_λ	$= 0.1$ ϵ/L	I_λ	$= 1.0$ ϵ/L	I_λ	$= 10.0$ ϵ/L	I_λ
3000	0.3	7.80E-02	6.30E-16	1.87E-16	1.08E-14	3.21E-15	1.29E-13	3.83E-14	1.33E-12	3.95E-13	1.26E-11	3.75E-12
	0.6	6.10E-01	4.92E-15	1.36E-13	8.47E-14	2.34E-12	1.01E-12	2.79E-11	1.04E-11	2.88E-10	9.88E-11	2.73E-09
	1.2	5.00E 00	4.04E-14	1.90E-12	6.94E-13	3.27E-11	8.27E-12	3.90E-10	8.53E-11	4.02E-09	8.10E-10	3.82E-08
	2.4	4.00E 01	3.23E-13	3.52E-12	5.55E-12	6.04E-11	6.62E-11	7.20E-10	6.82E-10	7.43E-09	6.48E-09	7.[illegible]5E-08
	4.8	3.15E 02	2.54E-12	2.35E-12	4.37E-11	4.04E-11	5.21E-10	4.82E-10	5.37E-09	4.97E-09	5.10E-08	4.72E-08
6000	0.3	7.80E-02	3.35E-12	2.97E-09	1.83E-10	1.61E-07	9.93E-09	8.78E-06	4.61E-07	4.08E-04	1.33E-05	1.17E-02
	0.6	6.10E-01	2.62E-11	3.96E-08	1.43E-09	2.15E-06	7.76E-08	1.17E-04	3.60E-06	5.44E-03	1.04E-04	1.57E-01
	1.2	5.00E 00	2.15E-10	7.49E-08	1.17E-08	4.08E-06	6.36E-07	2.22E-04	2.95E-05	1.03E-02	8.52E-04	2.97E-01
	2.4	4.00E 01	1.72E-09	5.09E-08	9.36E-08	2.77E-06	5.09E-06	1.51E-04	2.36E-04	6.99E-03	6.81E-03	2.02E-01
	4.8	3.15E 02	1.35E-08	2.07E-08	7.37E-07	1.12E-06	4.01E-05	6.11E-05	1.86E-03	2.84E-03	5.37E-02	8.18E-02
9000	0.3	7.80E-02	3.00E-10	3.82E-06	9.82E-09	1.25E-04	3.18E-07	4.0 E-03	1.13E-05	1.44E-01	4.70E-04	5.98E 00
	0.6	6.10E-01	2.35E-09	1.34E-05	7.68E-08	4.39E-04	2.48E-06	1.42E-02	8.87E-05	5.07E-01	3.67E-03	2.10E 01
	1.2	5.00E 00	1.92E-08	1.30E-05	6.29E-07	4.27E-04	2.04E-05	1.38E-02	7.27E-04	4.93E-01	3.01E-02	2.04E 01
	2.4	4.00E 01	1.54E-07	6.35E-06	5.03E-06	2.08E-04	1.63E-04	6.72E-03	5.81E-03	2.40E-01	2.41E-01	9.95E 00
	4.8	3.15E 02	1.21E-06	2.18E-06	3.97E-05	7.14E-05	1.28E-03	2.31E-03	4.58E-02	8.25E-02	1.90E 00	3.42E 00
12000	0.3	7.80E-02	2.36E-09	1.14E-04	1.06E-07	5.11E-03	3.81E-06	1.84E-01	1.29E-04	6.23E 00	4.24E-03	2.05E 02
	0.6	6.10E-01	1.84E-08	2.06E-04	8.27E-07	9.22E-03	2.98E-05	3.32E-01	1.01E-03	1.12E 01	3.32E-02	3.70E 02
	1.2	5.00E 00	1.51E-07	1.43E-04	6.78E-06	6.42E-03	2.44E-04	2.31E-01	8.27E-03	7.83E 00	2.72E-01	2.57E 02
	2.4	4.00E 01	1.21E-06	5.90E-05	5.42E-05	2.65E-03	1.95E-03	9.53E-02	6.62E-02	3.23E 00	2.18E 00	1.06E 02
	4.8	3.15E 02	9.52E-06	1.86E-05	4.27E-04	8.36E-04	1.54E-02	3.01E-02	5.21E-01	1.02E 00	1.71E 01	3.35E 01
15000	0.3	7.80E-02	2.03E-09	2.18E-04	2.77E-07	2.98E-02	1.50E-05	1.61E 00	5.98E-04	6.42E 01	2.24E-02	2.41E 03
	0.6	6.10E-01	1.59E-08	2.64E-04	2.17E-06	3.60E-02	1.17E-04	1.95E 00	4.68E-03	7.77E 01	1.75E-01	2.91E 03
	1.2	5.00E 00	1.30E-07	1.50E-04	1.78E-05	2.05E-02	9.60E-04	1.11E 00	3.83E-02	4.43E 01	1.44E 00	1.66E 03
	2.4	4.00E 01	1.04E-06	5.61E-05	1.42E-04	7.66E-03	7.68E-03	4.14E-01	3.07E-01	1.65E 01	1.15E 01	6.20E 02
	4.8	3.15E 02	8.19E-06	1.69E-05	1.12E-03	2.30E-03	6.05E-02	1.25E-01	2.41E 00	4.97E 00	9.05E 01	1.86E 02

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